
THERMODYNAMICS OF TURBOCOMPRESSORS

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Compression in turbocompressor

The characteristic feature of compression in a turbocompressor is the continuity of the process of transformation of work into pressure and internal energy of a working gas. The basic requirement is to increase the pressure, or achieve the required compression ratio (see **Equation 610**), with a minimum increase in temperature, which increases significantly, especially if the compression is not cooled.

– **610:** –
 Compression ratio of
 compressor

$$\varepsilon = \frac{p_e}{p_i}$$

p [Pa] pressure; ε [1] compression ratio. The index i indicates the state at the inlet to the turbocompressor, the index e indicates the state at the outlet of the turbocompressor.

When investigating a compression process, it is necessary to distinguish between single- and multi-stage compression. Both compressions require a little different approach to understanding the causes of losses and an approach to determining measures to reduce them.

Adiabatic compression

The adiabatic compression computational model is used in cases where no significant effect of heat exchange with the turbocompressor surroundings is expected.

Definition of ideal
 adiabatic
 compression

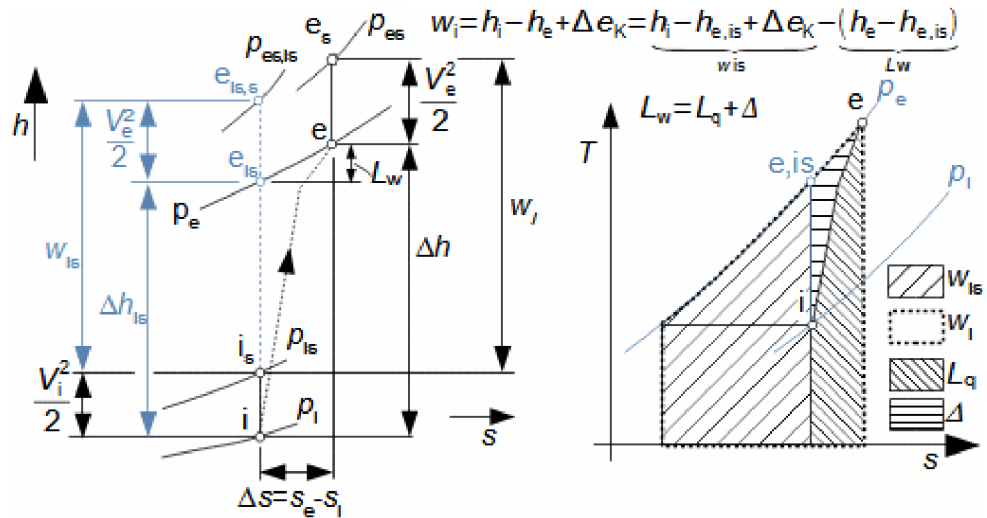
The ideal internal work of adiabatic compression corresponds to that of isentropic compression. In ideal compression, the same outlet velocity is usually expected as in actual compression. This means that the actual machine must have a slightly larger flow area than an ideal machine because the heat loss increases the specific volume of the working gas.

Actual adiabatic
 compression

A characteristic feature of real compression is the so-called additional losses Δ . These losses are equivalent to the extra work that has to be added due to the increase in the specific volume of the working gas caused by the loss heat. **Figure 118** (p. 4) shows an example of compression in a compressor or its stage in $h-s$ and $T-s$ charts. While in the $h-s$ chart only the losses as a whole can be distinguished, in the $T-s$ chart the individual types of losses can be distinguished.

118:

Adiabatic compression in h - s and T - s chart

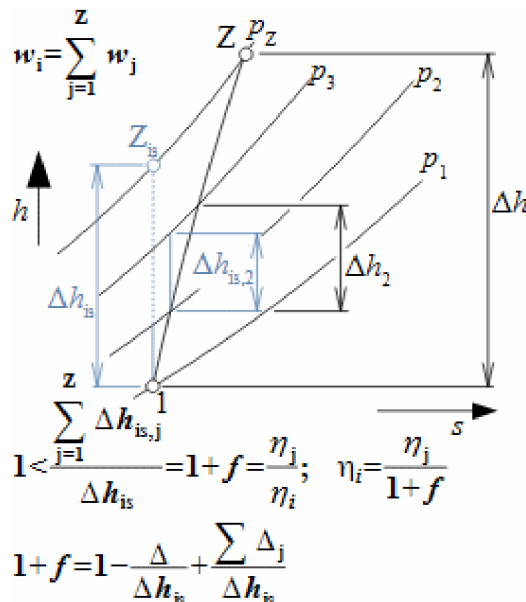


L_q [$J \cdot kg^{-1}$] loss heat ([Škorpík, 2024]); Δe_K [$J \cdot kg^{-1}$] kinetic energy difference between inlet and outlet (usually insignificant difference); h [$J \cdot kg^{-1}$] enthalpy; Δh [$J \cdot kg^{-1}$] enthalpy difference; Δh_{is} [$J \cdot kg^{-1}$] enthalpy difference at isentropic compression; L_w [$J \cdot kg^{-1}$] internal losses in compressor (extra work input to stage compared to is. compression); s [$J \cdot kg^{-1} \cdot K^{-1}$] entropy; T [K] absolute temperature; V [$m \cdot s^{-1}$] velocity; v [$m^3 \cdot kg^{-1}$] specific volume; w_i [$J \cdot kg^{-1}$] internal work; w_{is} [$J \cdot kg^{-1}$] internal work at is. compression; Δ [$J \cdot kg^{-1}$] additional losses. The index $_{is}$ denotes the isentropic compression states, the index $_s$ the stagnation state. The T - s chart is constructed at insignificant Δe_K . These equations are derived in **Appendix 118**.

Analysis of multi-stage adiabatic compression

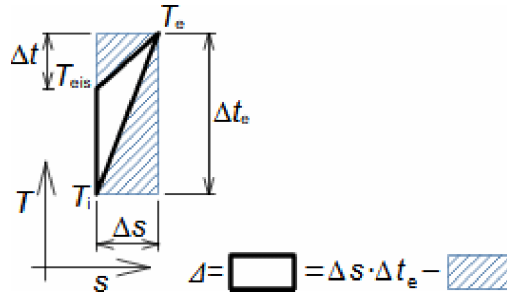
A typical characteristic of the internal efficiency of multi-stage compression η_i is that it is smaller than the mean internal efficiency of the individual stages η_j , see **Figure 121**. The cause is due to additional losses. Thus, it is clear that the internal losses in the compressor stage increase the work required in the following stage. The ratio of the mean value of the internal efficiency of the individual stages η_j to the internal efficiency measured between the first and the last stage η_i is called the preheat coefficient $1+f$, see **Problem 122** (p. 5).

121:



$1+f$ [1] preheat coefficient; Z [-] number of stages; Δ_j [$J \cdot kg^{-1}$] additional losses per stage; η_i [1] internal compression efficiency between points 1-Z. The index $_j$ indicates the j -th stage. The equations are derived for the assumption that all stages process the same enthalpy gradient and the compression is adiabatic. For clarity, the kinetic energy of the absolute velocity is not plotted in the figure. These equations are derived in **Appendix 121**.

- **Problem 122:** - The turbocompressor intakes air at a temperature of 15 °C and a pressure of 0,1013 MPa, the air at the turbocompressor outlet is 293 °C and the pressure is 0,802 MPa. Determine the additional losses, the preheat coefficient and the internal efficiency η_i . The turbocompressor has 12 stages. The compression is uncooled respectively consider adiabatic compression. The solution of this problem is shown in **Appendix 122**.

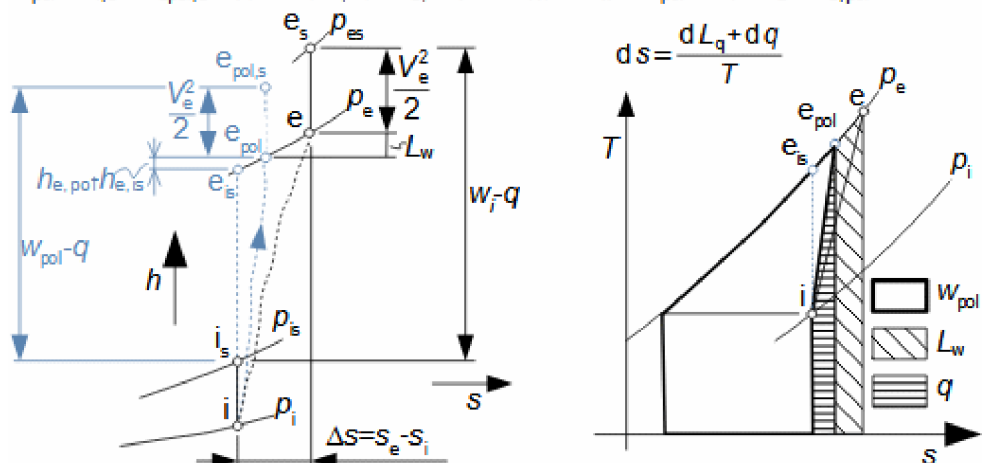


Using linear approximation of thermodynamic changes in $T-s$ chart to approximate magnitude of additional losses at compression: T [K]; t [°C]; s [$J \cdot K^{-1} \cdot kg^{-1}$]

Polytropic compression

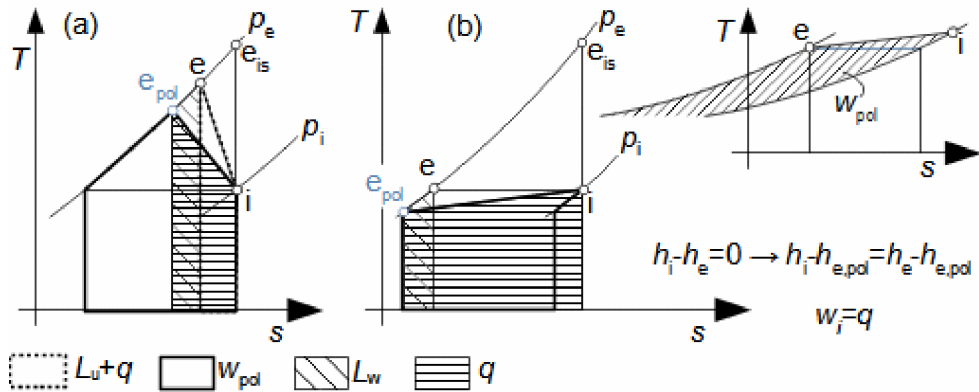
In some cases, compression is affected by heat exchange with the compressor surroundings. For example, when the compressor is purposely cooled, or when cryogenic gas that is heated by the surrounding is compressed. In such cases, the compression is similar to polytropic compression - the comparative ideal compression in this case is reversible polytropic compression. Polytropic compression is described by **Equations 687**. These equations can be derived from the equation of the first law of thermodynamics.

- **687:** - $w_{pol} = h_{i,s} - h_{e,pol,s} + q$; $w_i = (h_i - h_e) + q + \Delta e_{K,i}$; $L_w = w_{pol} - w_i = h_e - h_{e,pol}$
 Polytropic compression for case $q > 0$



q [$J \cdot kg^{-1}$] heat exchanged with surroundings. The index pol indicates reversible polytropic compression. In the figure, the case $q > 0$ (heat input - if $h_{e,pol} - h_{e,is} > 0$, then this is the sum of the heat input and the additional losses due to heat input). The $T-s$ chart is constructed when the difference in kinetic energies is insignificant.

– **688:** –
 Polytropic
 compression for case
 of cooled
 compression

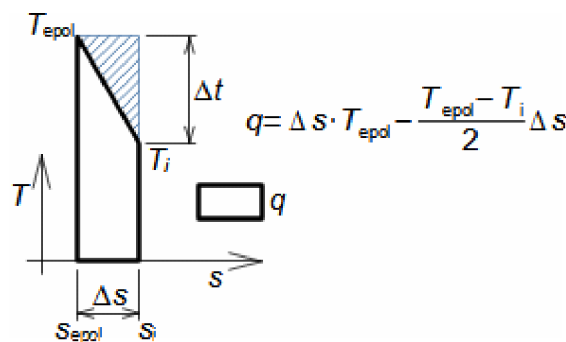


(a) case for $L_w < -q$; (b) case when $T_e = T_i$ (apparently isothermal compression - in this case temperature of cooling medium must be lower than temperature of working gas at inlet to compressor T_i). If $h_{e,pol} - h_{e,is} < 0$, then this is the sum of the heat rejected and the work saved due to the compression cooling. A T - s diagram is constructed when the difference in kinetic energies is insignificant.

Comparative
 reversible polytropic
 compressions

When creating energy balances for polytropic compression, it is necessary to define the work in reversible polytropic compression w_{pol} . Often the work of reversible isothermal compression is used as w_{pol} , especially if the compression is cooled, see **Problem 849**. In particular, the work of compression with heat input is compared with the work of isentropic compression w_{is} . In presenting the internal efficiencies, it is necessary to indicate which process has been selected as the comparison process in order that the efficiency value may have a telling value.

– **Problem 849:** – Find the internal isentropic, polytropic and isothermal efficiency of a turbocompressor that compresses dry air. The inlet air temperature is 14,34 °C and the outlet air temperature is 480,6 °C. The inlet pressure is atmospheric and the compression ratio is 23. The internal input power of the turbocompressor is 12,6 MW. The turbocompressor is equipped with a casing cooling with a capacity of 0,8 MW. The solution of this problem is shown in **Appendix 849**.



Using linear approximation of reversible polytropic compression in T - s chart to approximate state e_{pol} : T [K]; t [°C]; s [$J \cdot K^{-1} \cdot kg^{-1}$]; q [$kJ \cdot kg^{-1}$]

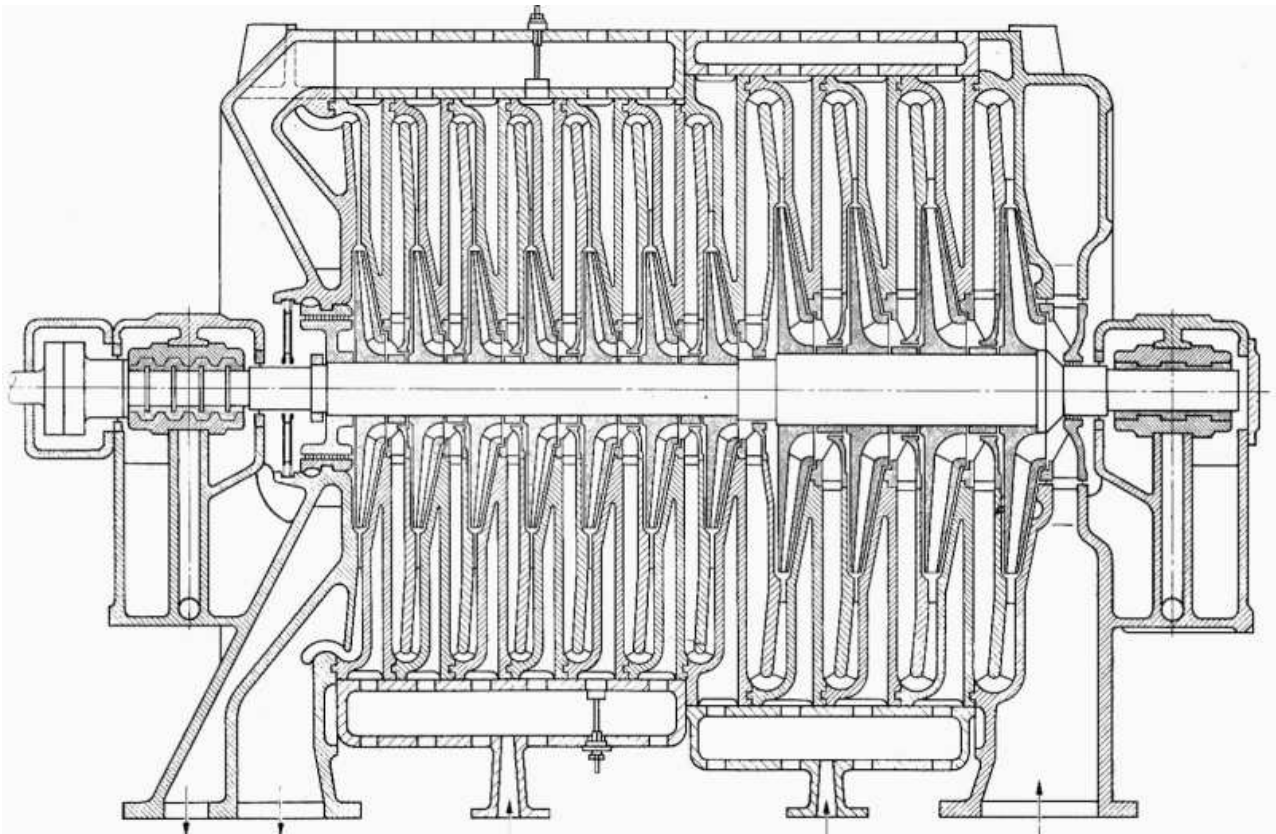
Turbocompressors cooling performance

*Compression cooling is the most effective way to reduce compressor internal work, with several methods to achieve it. The compressed gas during compression can be continuously cooled, either by **casing cooling** or by **injecting coolant** into the compressed gas. However, cooling can also be done discontinuously in stages by means of so-called **intercooling**. However, each cooling generates a new type of loss, so that **effective cooling** can only be done under certain conditions.*

~
Casing cooling of
turbocompressors

Casing cooling can be done on twin-casing compressors, with coolant flowing between the casings to cool the working gas inside, see **Figure 608**. Casing cooling compressors are complicated and expensive - channels are required and there is a risk of coolant leakage into the compressed gas at the dividing plane and vice versa.

608:



Turbocompressor with eleven radial stages and with casing cooling. Made by Demag.

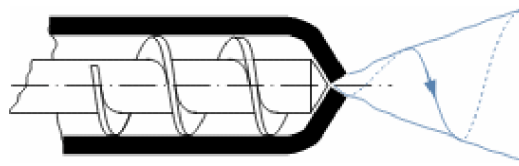
Examples of casing
colling applications

Casing cooling is a low efficiency method and so is used for compressors with low compression in one stage, for single stage compressors and blowers it is the only way to cool the compression. On the other hand, its low efficiency allows it to be used for cooling even moist air containing dust, provided that the compressor surface temperature does not drop below the dew point of the air.

~
*Principle of cooling
 by coolant injection*

The working gas flow is cooled by evaporation of the injected coolant. The rate of evaporation and therefore cooling depends, among other things, on the relative heat transfer surface of the coolant and the working gas, so the injection nozzles (**Figure 932**) are designed to have as much dispersion as possible. A certain distance is required for evaporation and therefore radial stages are preferable for injection cooling (coolant is injected at a point behind the stator blades towards the return passage to the next stage). In the case of axial stages, the gap between the stages would have to be increased at the injection point.

— **932:** —
*Injection nozzle for
 injection of coolant*



*Selection of the type
 of injection coolant
 and limits on its
 quantity*

The amount of coolant depends on the pressure, the required temperature and the composition of the resulting mixture after cooling. For example, if the compressed gas is air, only enough cooling water can be injected to keep the relative humidity of the air below 100 % after evaporation, otherwise water droplets will remain in the air. In ammonia compression, liquid ammonia is used, in nitrous gas compression, a low nitric acid solution is used, etc.

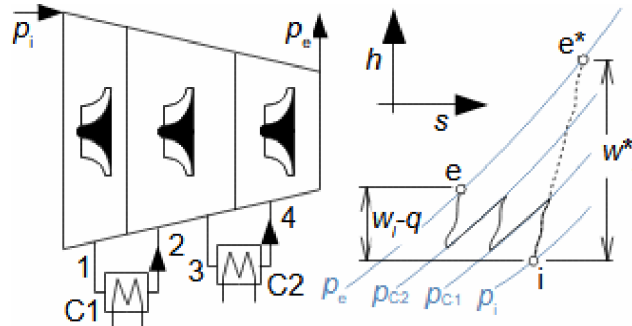
*Limitations on use of
 cooling by coolant
 injection*

The disadvantage of this method of cooling is that the compressor discharge gas contains some moisture. This means that the use of such gas is limited to applications where moisture in the gas is not a barrier to its use. In particular, these are processes where the vapours contained in the gas may condense. For example, when used in pneumatic actuators, etc.

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*Principle of
 compression with
 intercooling*

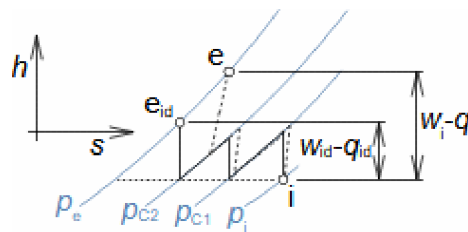
The intercooling method consists of taking the compressed gas behind the selected compressor stages outside the compressor to a heat recovery heat exchanger (most often consisting of finned tubes) where the gas is cooled by means of a coolant (usually water). For example, in the case of **Figure 671** (p. 9), where intercooling is implemented for a three-stage turbocompressor, the entire compression can be divided into 3 separate compressions and the internal work of the compressor can be calculated from the differences in enthalpies and heat rejection, see **Problem 612** (p. 9). The advantage of intercooling is also that the work of the individual stages and their working conditions are similar (velocity triangles, temperatures, etc.), but it must be taken into account that the specific volume of gas decreases between stages, therefore the first stage behind intercooling will have smaller inlet flow area than the outlet flow area of the previous stage.

– 671: –



C-intercoolers; e^* -end state of working gas at compressor outlet in case of compression without cooling. p_{C1} , p_{C2} [Pa] pressure before entering intercoolers; w_i^* [$J \cdot kg^{-1}$] internal work of compressor for case of compression without cooling.

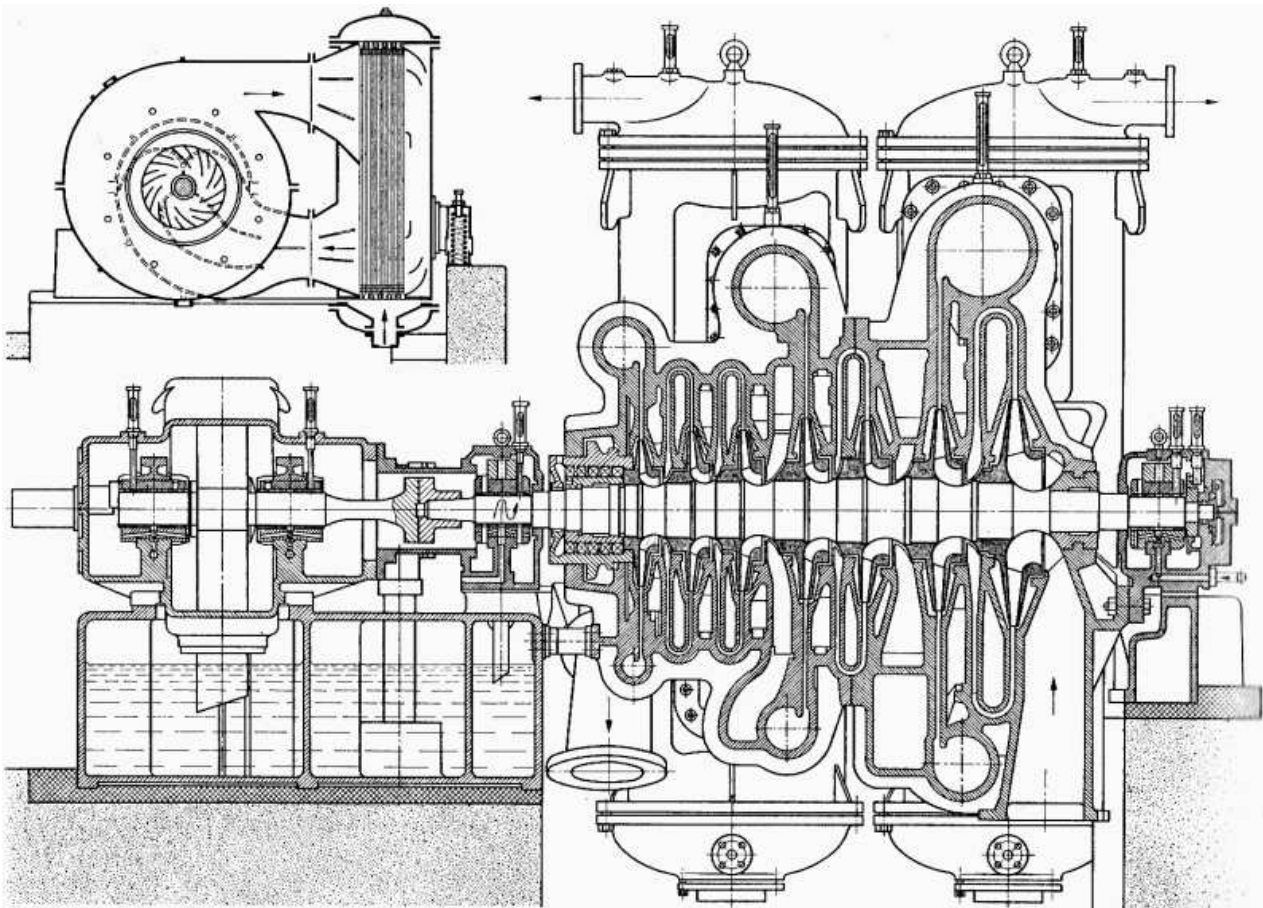
- **Problem 612:** – Find the internal efficiency of the turbocompressor that compresses the dry air. The inlet air temperature is 14,34 °C and the outlet air temperature is 156,6 °C. The inlet pressure is atmospheric and the compression ratio is 23. The internal input power of the turbocompressor is 10,6 MW. The turbocompressor is equipped with two intercoolers at a pressure levels of 0,7 MPa and 1,4 MPa. The cooling capacity of the coolers is 6,5 MW. The solution of this problem is shown in **Appendix 612**.



Compressor design with intercooling

Figure 840 (p. 10) shows an example of a turbocompressor design with seven radial stages and two intercoolers, placed behind the second and fourth stages. Note that the intercoolers must also be removed before dismantling the compressor top case (see cross section). There are other design arrangements for easier removal of the compressor - connecting the inlet and outlet of the intercooler through the lower case. In the case of multicasing turbocompressors, intercooling can be installed in the interconnection between the individual casings.

840:



Seven-stage turbocompressor with two intercoolers. Made by Escher Wyss.

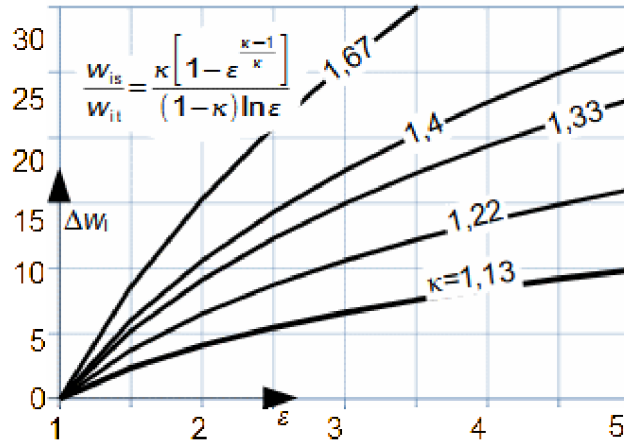
Limitations on use of intercooling

This method of cooling is accompanied by greater design and investment requirements (cooling equipment must be purchased in addition to the compressor) and is therefore usually carried out only from a certain size of turbocompressor or there must be other than economic reasons, such as safety (for flammable gases), gas stability (molecules can dissociate at higher temperatures), etc.

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Cooling effectiveness

Cooling does not always mean a significant reduction in power input. Any cooling affects the thermodynamics of compression (cooling increases the pressure loss, by friction of the fluid against the heat transfer surfaces and by the formation of vortices when injecting coolant, etc.), so there is always a limit to the effectiveness of cooling. As can be seen by comparing the work of isentropic compression with that of isothermal compression, which corresponds to the compression at perfect cooling in the chart in **Figure 637** (p. 11). For example, it is clear from the chart or equation that if the increase in the work of air compression due to pressure losses were 10%, then cooling would be positively significant at compression ratios as low as 2, at methane compression as high as $\varepsilon=5$, etc. At actual compressions, the work savings are much less, so it pays to start cooling from higher compression ratios than the chart shows.

637:
Difference between
isentropic and
isothermal
compression work



w_{it} [$J \cdot kg^{-1}$] work of isothermal compression; Δw_i [%] maximum theoretical work saving due to cooling, $\Delta w_i = (w_{is} \cdot w_{it}^{-1} - 1)100$; κ [1] heat capacity ratio of working gas ($\kappa=1,13$ for example CH_4 , $\kappa=1,22$ for example C_2H_4 , $\kappa=1,33$ for example H_2O steam, $\kappa=1,4$ for example air, $\kappa=1,67$ for example He). The derivation of the equation is shown in **Appendix 637**.

Thermodynamic design of turbocompressor stage

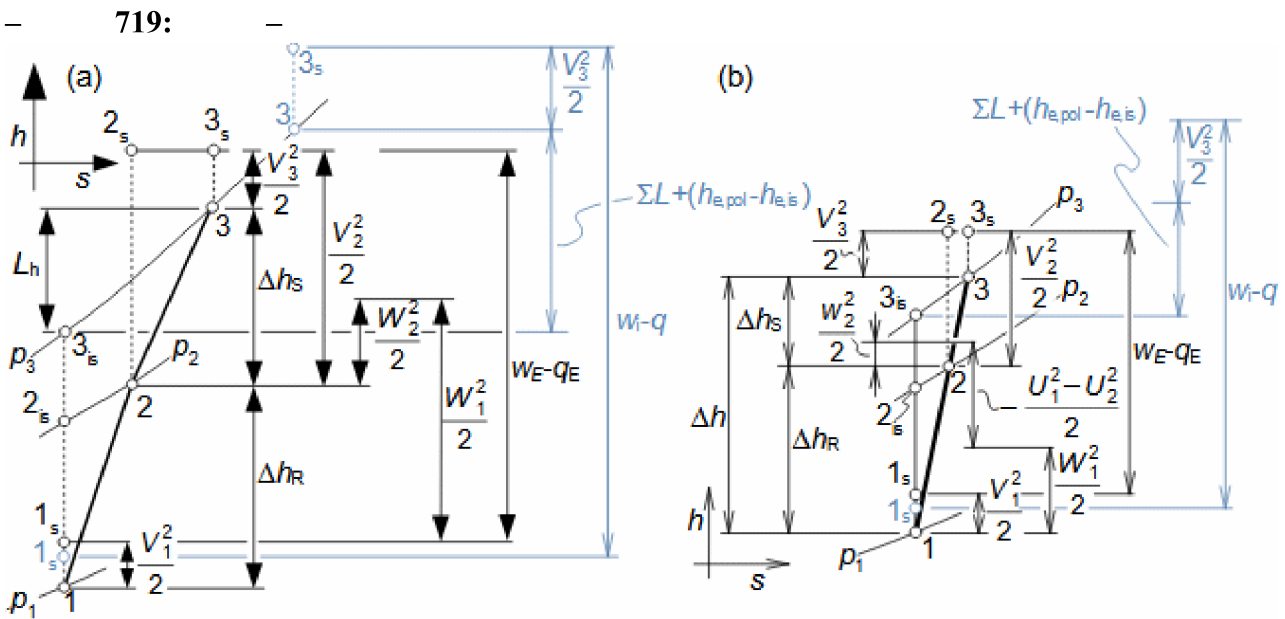
For thermodynamic calculations of the compressor stage, findings from previous articles in these proceedings can be utilized (*Turbomachinery*). Here, only special knowledge on compressor stage thermodynamics is summarized and supplemented: **selecting stage type; h-s stage charts; prediction of Euler work; turbocompressor blades**. Recommended values of dimensionless coefficients for the design of individual stages are given in [Japikse, 1997, p. 1-3].

~
Selecting stage
compressor type

The basic types of compressor stages are reaction axial stage and radial stage. Occasionally, a diagonal design of a single-stage compressor can be encountered. Most stationary applications fall into the radial stage range, only when higher mass flow of about $15 \text{ m}^3 \cdot \text{s}^{-1}$ and above are required is the compressor designed as an axial stage, because from such mass flows the efficiency of axial turbocompressors is already higher than radial ones. On the other hand, the intercooling is better implemented with radial stages, which more than compensates for the lower efficiency of the individual stages.

~
h-s charts

Figure 719 (p. 12) shows the *h-s* chart of the axial and radial compressor stage at the investigated radius. At each radius under investigation, the effect of the heat exchanged q_E on the Euler work can be different depending on the type of cooling and the heat from the rotor friction loss. The energy balance of the whole stage is shown in blue.

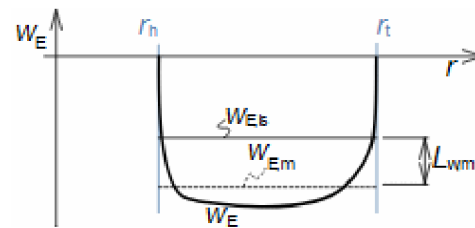


h-s charts of compressors stage at investigated radius r : (a) axial stage; (b) radial stage. L_h [$J \cdot kg^{-1}$] profile losses; q_E [$J \cdot kg^{-1}$] heat exchanged with surrounding of investigated streamline; ΣL [$J \cdot kg^{-1}$] internal losses of stage, sum of all losses in stage. The index $_1$ indicates the condition before the rotor blade cascade.

~
Euler work

Figure 609 shows the expected Euler work w_E of the axial stage compressor. The profile losses are highest at the blade edges and therefore the necessary pressure increase cannot be achieved at these points (the necessary Euler work is growing beyond all limits) and, on the contrary, flow separation and even reverse flow losses can be expected. The differences in Euler work between the stream core and the blade edges are even greater in the case of straight blades of axial stages and are therefore not much used in compressors.

609:
Euler work profile of compressor axial stage

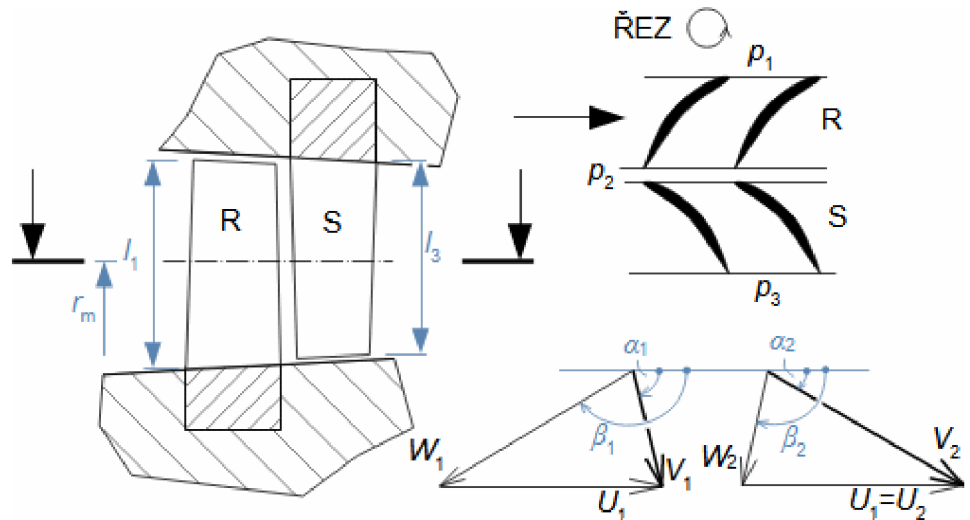


r [m] radius of stage; $w_{E,m}$ [$J \cdot kg^{-1}$] mean value of Euler work of stage. The index $_h$ denotes the foot radius of the blades, the index $_t$ the radius at the blade tips.

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Turbocompressor blades

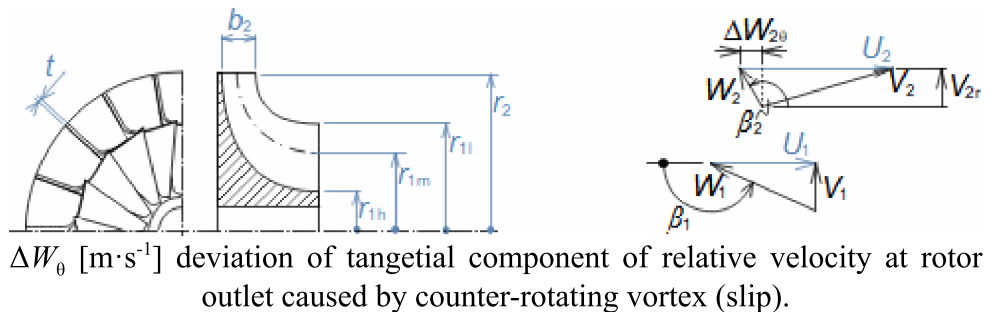
Axial or conical compressor stages contain twisted blades, but there are also cases with straight blades. The *h-s* chart for a radial stage can also be used in the design of a conical compressor stage, in which the radius of the stage, respectively the length of the blades, decreases due to the decreasing specific volume, see **Figure 1101** (p. 13) (see the calculation of a conical stage in the article Internal losses of turbomachines and their influence on turbomachine calculation). Due to the thin blades, shroud cannot be used in compressor stages.

- **1101:** –
Velocity triangle of axial compressor stage



R-rotor blade cascade; S-stator blade cascade. l [m] blade length; r_m [m] mean radius; U [$\text{m}\cdot\text{s}^{-1}$] blade speed; V [$\text{m}\cdot\text{s}^{-1}$] absolute velocity; W [$\text{m}\cdot\text{s}^{-1}$] relative velocity; α [$^\circ$] angle of absolute velocity; β [$^\circ$] angle of relative velocity. The velocity triangle is drawn for a mean radius and a reaction of 0,5.

- **Problem 726:** – Make a basic design of the rotor dimensions of a single-stage radial turbocompressor with axial inlet. The rotor blades have a radial direction at the outlet, see figure. The dry air parameters at the rotor inlet are: 101,33 kPa, 15 °C. The required pressure from the stator blade cascade is 0,44 MPa. The required mass flow rate is 0,7225 $\text{kg}\cdot\text{s}^{-1}$. The solution of this problem is shown in **Appendix 726**.

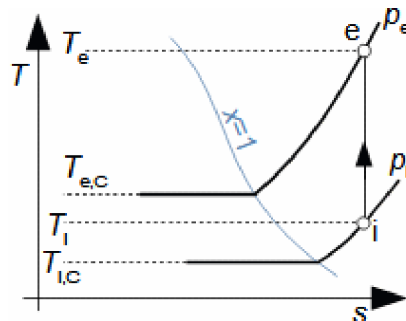


ΔW_{θ} [$\text{m}\cdot\text{s}^{-1}$] deviation of tangential component of relative velocity at rotor outlet caused by counter-rotating vortex (slip).

Change in relative humidity at compression

Compressing moist air increases the pressure of the gases and the steam pressure contained in the air. In adiabatic compression, the steam content will always be in a superheated state at the end of compression, even in the case of saturated air compression. This means that the relative humidity at the end of compression will always be lower than at the beginning and therefore condensation of the steam in the air cannot occur. However, at higher pressures the condensation temperature of the steam in the air will also rise from the initial absolute temperature $T_{i,C}$ to $T_{e,C}$, see **Figure 1050** (p. 14).

1050: T-s chart of steam compression in air



p [Pa] partial pressure of steam in air; T_c [K] absolute temperature of condensation of steam in air at pressure at start of compression (index i) and at end of compression (index e); x [1] dryness of steam. The figure shows the case of isentropic compression.

Amount of condensate rejected from compressed and cooled moist air

Condensation of steam in compressed air can occur, for example, in the piping during its distribution to consumers or during its cooling in compressor intercoolers or compressed air storage tanks, etc. In these cases, it is usually assumed that the compressed moist air will be cooled to ambient temperature, i.e. the suction temperature of the compressor T_i . The task of the engineer or designer is therefore to determine whether condensate will be rejected at this temperature and in what quantity according to **Equation 1049**. This equation was derived assuming that the moist air is cooled to the inlet temperature, if the resulting cooling temperature is lower, the amount of condensate rejected will be greater.

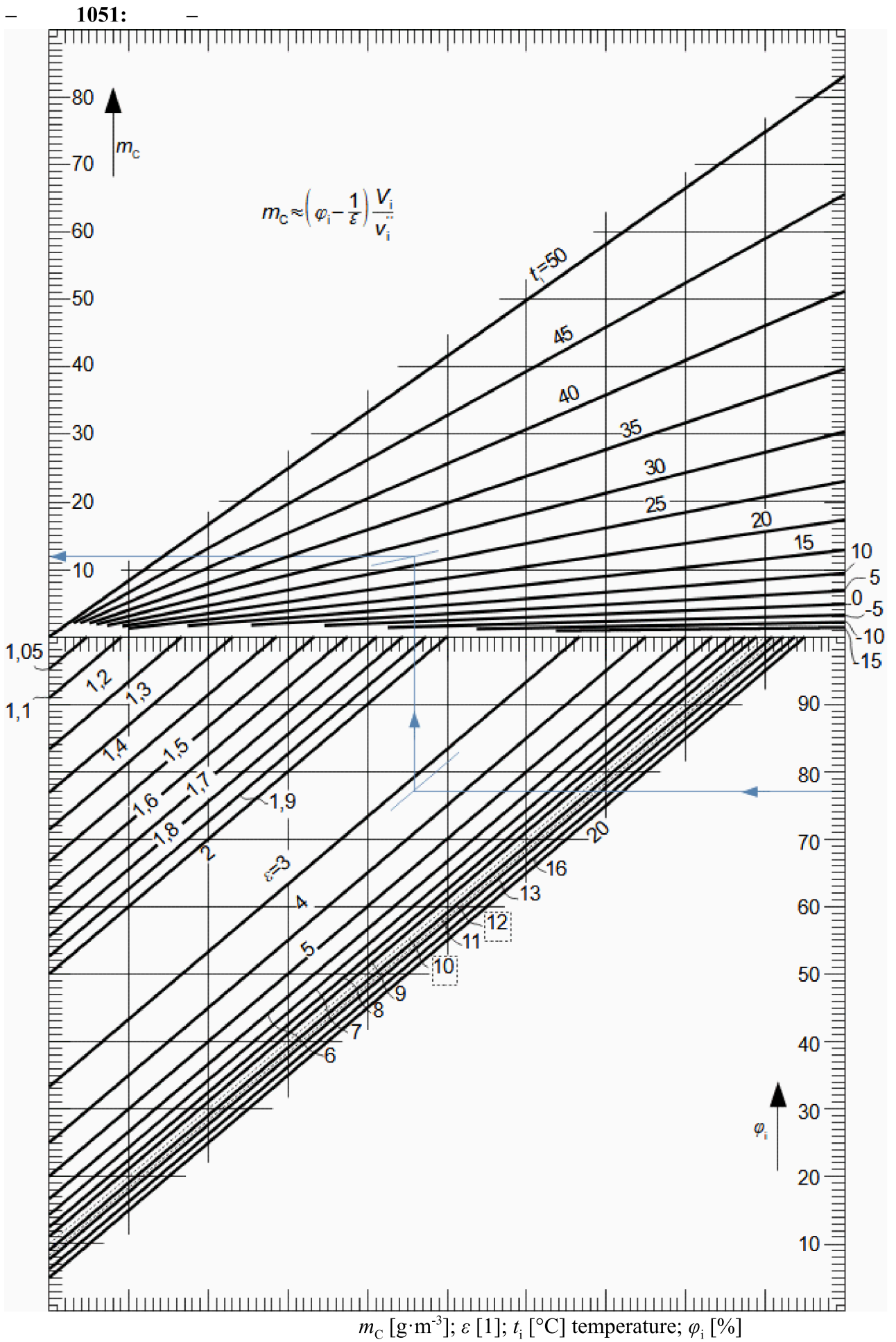
1049:

$$m_c \approx \left(\varphi_i - \frac{1}{\varepsilon} \right) \frac{V_i}{v_i}$$

m_c [kg] amount of condensate rejected from compressed and cooled moist air back to temperature t_i (negative value means that relative humidity of air at end of compression and after cooling φ_e will be less than 1 and therefore no condensation will occur); V_i [m³] volume of compressed air measured at inlet; v_i [m³·kg⁻¹] specific volume of saturated steam at inlet temperature t_i ; φ [1] relative humidity of air. The derivation of this equation is shown in **Appendix 1049**.

Nomogram for approximate determination of amount of condensate rejected

The specific volume of saturated steam in **Equation 1049** is a function of temperature $v''=f(t)$, therefore a nomogram can be constructed to determine the amount of condensate rejected from compressed and cooled moist air as a function of inlet temperature, see **Nomogram 1051** (p. 15).



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